





# **UNIFIED INTERNATIONAL MATHEMATICS OLYMPIAD (UPDATED)**

CLASS - 10

Question Paper Code : 4P104

### KEY

1	2	3	4	5	6	7	8	9	10
А	А	С	С	С	С	А	В	А	С
11	12	13	14	15	16	17	18	19	20
В	В	С	D	D	С	В	D	А	В
21	22	23	24	25	26	27	28	29	30
С	Α	D	А	С	А	В	В	D	D
31	32	33	34	35	36	37	38	39	40
A,B	A,B,D	C,D	A,C	B,C	D	D	D	C, D	D
41	42	43	44	45	46	47	48	49	50
D	В	D	С	В	А	С	В	С	В

## SOLUTIONS

#### MATHEMATICS - 1 (MCQ)

- 01. (A) If a circle inscribed in a quadrilateral then sum of opposite angles made at the centre are supplimantary
  - ∴ 115° + ∠COD = 180°

∠COD = 180° - 115° = 65°

02. (A)  $210 = 5 \times 7 \times 2 \times 3$ 

 $65 = 5 \times 13$ 

... HCF of 210 & 65 = 5 Given  $199 \times 5 + 55y = 5$ 

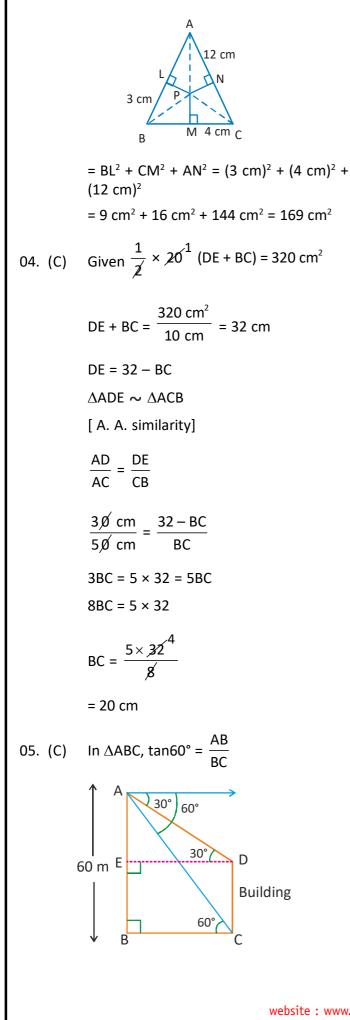
$$199 \times 5 - 5 = -55y$$

$$\frac{198^{18} \times \cancel{5}}{-55} = y$$

*y* = –18

$$\therefore y^2 = (-18)^2 = 324$$

03. (C) Construction :- Join PA, PS & PC  $AL^{2} + BM^{2} + CN^{2} = AP^{2} - PL^{2} + BP^{2} - PM^{2} + CP^{2} - PN^{2}$  $= BP^{2} - PL^{2} + CP^{2} - PM^{2} + AP^{2} - PN^{2}$ 



$$\sqrt{3} = \frac{60 \text{ mts}}{\text{BC}}$$

$$BC = \frac{60 \text{ mts}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= 20\sqrt{3} \text{ mts}$$

$$\ln \Delta AED, \tan 30^{\circ} = \frac{AE}{ED}$$

$$\frac{1}{\sqrt{3}} = \frac{AE}{20\sqrt{3} \text{ mts}}$$

$$[\because ED = BC = 20\sqrt{3} \text{ m}]$$

$$\therefore AE = \frac{20\sqrt{3} \text{ m}}{\sqrt{3}} = 20 \text{ m}$$

$$\therefore BE = AB - AE = 40 \text{ m}$$

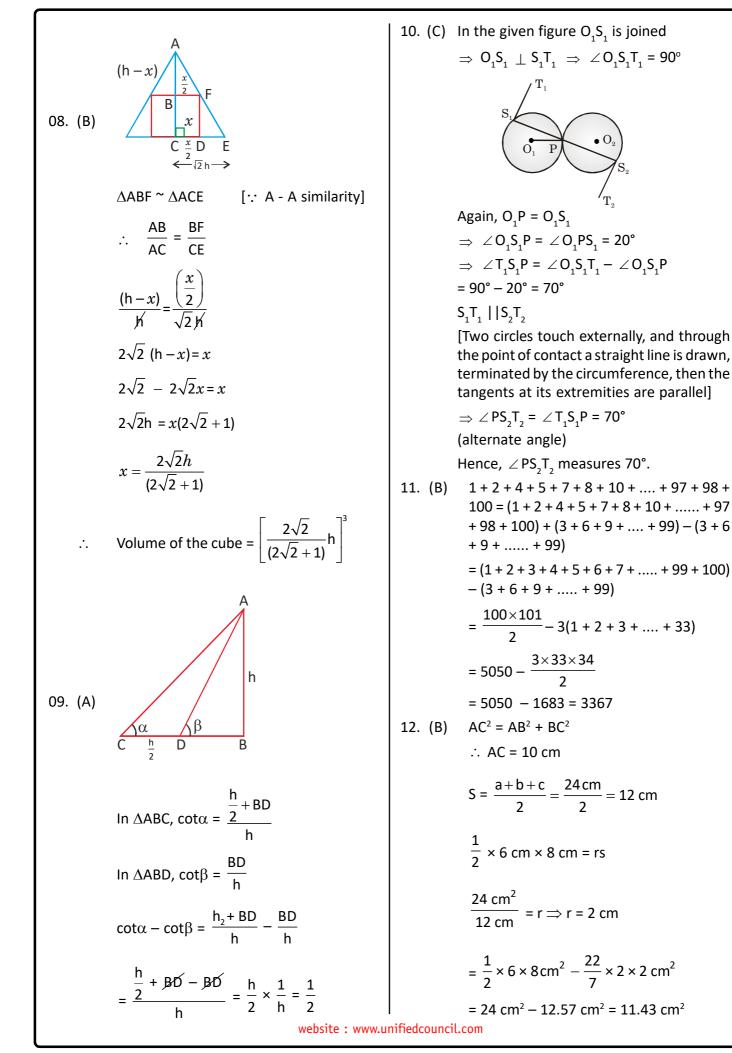
$$\therefore CD = BE = 40 \text{ m}$$
O6. (C) ABCD is a square of 7cm each side.  
Area of the shaded region  

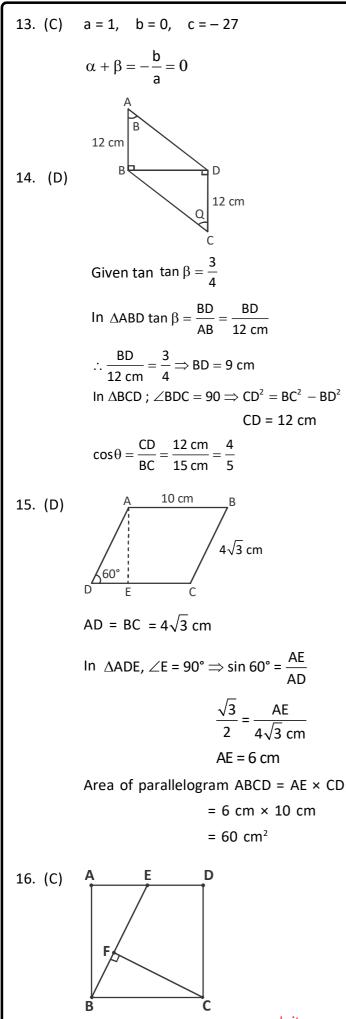
$$= 7 \times 7cm^{2} - \frac{x}{360^{\circ}} \times \pi r^{2}$$

$$= 49cm^{2} - \frac{90^{\circ}}{360^{\circ}} \times \frac{22}{7} \times 7 \times 7$$

$$= 49cm^{2} - 38.5 \text{ cm}^{2} = 10.5 \text{ cm}^{2}$$
O7. (A)  $\frac{a_{1}}{a_{2}} = \frac{5}{3}, \frac{b_{1}}{b_{2}} = -\frac{15}{-9} = \frac{5}{3}$ 

$$\therefore \frac{a_{1}}{a_{2}} = \frac{b_{1}}{b_{2}} = \frac{c_{1}}{c_{2}} \Rightarrow \text{ coinciding lines}$$





	Since $\angle$ EBA = $\angle$ FCB and $\angle$ FBC = $\angle$ AEB, we have					
	$\triangle ABE \sim \triangle FCB.$ $\frac{AB}{FC} = \frac{BE}{CB} = \frac{EA}{BF}$ ,					
	$\frac{2}{FC} = \frac{\sqrt{5}}{2} = \frac{1}{BF}$					
	From those two equations, we find that					
	$CF = \frac{4}{\sqrt{5}}$ and $BF = \frac{2}{\sqrt{5}}$ . Now rthat we					
	have BF and CF, we can find the area of					
	the bottom triangle $\Delta CFB$ :					
	$\frac{1}{2} \cdot \frac{4}{\sqrt{5}} \cdot \frac{2}{\sqrt{5}} = \frac{4}{5}.$					
	The area of left triangle $\Delta {\sf BEA}$ is					
	$\frac{1}{2}$ .2.1=1. The area of the square is 4.					
	Thus, the area of the remaining					
	quadrilateral is $4 - 1 - \frac{4}{5} = \frac{11}{5}$ , and the answer is C.					
17. (B)	Let the three terms of am AP be a – d, a, a + d					
	Given a – d + a + a + d = $\sqrt{567}$					
	$3a = 9\sqrt{7}$					
	$a = 3\sqrt{7}$					
	Given $(3\sqrt{7}-d)(3\sqrt{7})(3\sqrt{7}+d) = 168\sqrt{7}$					
	$(3\sqrt{7}-d)(3\sqrt{7}+d) = \frac{168\sqrt{7}}{3\sqrt{7}} = 56$					
	$63 - d^2 = 56 \implies 63 - 56 = d^2$					
	$d = \pm \sqrt{7}$					
	If $d = \sqrt{7}$ , $a = 3\sqrt{7}$ then $a - d = 3\sqrt{7} - \sqrt{7} = 2\sqrt{7}$					
	a = 3√7					
	$a + d = 4\sqrt{7}$					
d :	$= -\sqrt{7} a = 3\sqrt{7}$ then $a - d = 3\sqrt{7} - (-\sqrt{7}) = 4\sqrt{7}$					
	a + d = $3\sqrt{7}$ + $\left(-\sqrt{7}\right)$ = $2\sqrt{7}$ = $\sqrt{28}$					

18. (D) Distance from origin for  $\left(\frac{13}{2}, 0\right)$  $=\sqrt{\left(\frac{13}{2}\right)^2-0^2}=\frac{13}{2}$ Distance from origin to  $\left(-6, \frac{5}{2}\right)$  $=\sqrt{\frac{25}{4}+36}=\frac{13}{2}$ The point in option (D) lies on the circle. ÷. Given sin  $\theta$  + cos  $\theta$  =  $\sqrt{3}$ 19. (A) Squaring on both sides  $\sin^2 \theta + \cos^2 \theta + 2\sin \theta \cos \theta = 3$  $\therefore$  1 + 2 sin  $\theta$  cos  $\theta$  = 3  $2\sin\theta\cos\theta = 3-1$  $\therefore \sin \theta \cos \theta = \frac{1}{1} = 1$  $\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$  $=\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}=\frac{1}{1}=1$ 20. (B) Given  $S_{20} = \frac{20^{10}}{2} [2a + 19d] = 40$ 2a + 19d = 4..... (1) Given  $S_{40} = \frac{40^{20}}{2} [2a + 39d] = 20^{1}$ 2a + 39d = 1..... (2) eq (2) - (1)  $\Rightarrow$  20d = -3  $d = -\frac{3}{20}$  $2a - \frac{57}{20} = 4$ ..... (1)  $2a = 4 + \frac{57}{20} = \frac{137}{20} \implies a = \frac{137}{40}$  $S_{60} = \frac{60}{2} [2a + 59d]$  $=30\left[\frac{137}{20}-\frac{177}{20}\right]=30\left[-\frac{40}{20}\right]$  $S_{60} = -60$ 

21. (C) Let speed of the boat in still water be 'x' and speed of the stream be 'y' Given  $\frac{100}{x+y} + \frac{30}{x-y} = 6$  hours Let  $\frac{1}{x+y} = a$  and  $\frac{1}{x-7} = 6$ 100a + 30b = 6..... (1) Given  $\frac{75}{x+y} + \frac{75}{x-y} = 8$ 75a + 75b = 8 ..... (2) Eq (1)  $\times$  3  $\Rightarrow$  30/0a + 90b = 18 Eq (2) × 4  $\Rightarrow$  300a + 300b = 32 (-) (-) (-) +210h = +14 $b = \frac{14^{2^{1}}}{210_{30}}$  $100a + 30^2 \times \frac{1}{15} = 6$ 100a = 4 $a = \frac{4}{100} = \frac{1}{25}$  $\therefore$  a = x + y = 25 ..... (3) : b = x - y = 15..... (4) eq(3) + (4) 2x = 40x = 20 kmph 22. (A) Given  $\cos \theta = 1 - \cos^2 \theta = \sin^2 \theta$  $\sin^{12}\theta + 3\sin^{10}\theta + 3\sin^{8}\theta + \sin^{6}\theta = (\sin^{4}\theta)$ *.*•.  $(\theta)^3 + 3\sin^8\theta \sin^2\theta + 3\sin^4\theta \sin^4\theta + (\sin^2\theta)$  $\theta$ )<sup>3</sup> =  $(\sin^4 \theta + \sin^2 \theta)^3$ =  $(\cos^2 \theta + \cos \theta)^3$  $= 1^3 = 1$ 

23. (D) Area of the path =  $\frac{3}{5} \times 100 \times 60^{12} \text{ m}^2$  $= 3600 \text{ m}^2$ Let width of the path be x metres Total area = (100 + 2x)(60 + 2x).... = 6000 + 3600  $6000 + 200x + 120x + 4x^2 = 9600$  $\Rightarrow$  $4x^2 + 320x = 3600$  $x^2 + 80x = \frac{3600}{4}900$  $x^2 + 90x - 10x - 900 = 0$ x(x + 90) - 10(x + 90) = 0 $\therefore$  x = -90 (or) x = 10 Width of the path = (x) = 10 m *.*. 24. (A) The given equations are  $\frac{1}{2(2x+3y)} + \frac{12}{7(3x-2y)} = \frac{1}{2}$  .....(i)  $\frac{7}{(2x+3y)} + \frac{4}{(3x-2y)} = 2$  .....(ii) Putting  $\frac{1}{(2x+3y)}$  = u and  $\frac{1}{(3x-2y)}$  = v, the given equations become  $\frac{u}{2} + \frac{12v}{7} = \frac{1}{2} \implies 7u + 24v = 7$  .....(iii) and, 7u + 4v = 2.....(iv) On subtracting (iv) from (iii), we get  $20v = 5 \Longrightarrow v = \frac{5}{20} = \frac{1}{4}$  $\Rightarrow \frac{1}{(3x-2y)} = \frac{1}{4} \quad \left| \because \mathsf{v} = \frac{1}{(3x-2y)} \right|$  $\Rightarrow$  3x - 2y = 4 .....(v) Putting v =  $\frac{1}{4}$  in (iii), we get  $7u + 24 \times \frac{1}{4} = 7 \Longrightarrow 7u = (7 - 6) = 1 \Longrightarrow u = \frac{1}{7}$  $\Rightarrow \frac{1}{(2x+3y)} = \frac{1}{7} \qquad \because u = \frac{1}{(2x+3y)}$  $\Rightarrow 2x + 3v = 7$ .....(vi) Multiplying (v) by 3, (vi) by 2 and adding the results, we get  $13x = 26 \Rightarrow x = 2$ 

25. (C) Side of square = HCF of length and breadth of the room. 1763 cm & 1927 cm HCF = 41 cm1783) 1927 (1 1763 164) 1763 (10 1640 123) 164 (1 123 41) 123 (3 123 0 Least number of square tiles = ....  $\frac{1763^{43} \text{ cm} \times 1927^{47} \text{ cm}}{41_1 \text{ cm} \times 41_1 \text{ cm}}$ = 2021 26. (A) Given LCM + HCF = 1,94,292 (1) LCM - HCF = 1,93,788 (2) (-) (-)2 LCM = 388080  $LCM = \frac{388080}{2} = 1,94,040$ 1,94,040 + HCF = 1,94,292HCF = 1,94,292 - 1,94,040 = 252 But product of two numbers = LCM × HCF  $2520 \times x = 194040 \times 252$  $x = \frac{194040 \times 252}{2520} = 19404$ 27. (B) Let O be the position of the bird, B be the position of the boy and FG be the building at which G is the position of the girl Let OL, BF and GM, OL Then, BO = 100 m,  $\angle OBL = 30^\circ$ ,

FG = 20 m and  $\angle$ OGM = 45°.

29. (D) 
$$x^{6} - 3x^{4} + 3x^{2} - 1$$
  
 $= (x^{7})^{3} - 3(x^{2})^{2} + 3$   
 $= (x^{2} - 1)^{3} = (x + 1)^{3}$   
 $= (x^{2} - 1)^{3} = (x^{2} + 1)^$ 

$$= (x^{2})^{3} - 3(x^{2})^{2} + 3(x^{2})(1) - 1^{3}$$

$$= (x^{2} - 1)^{3} = (x + 1)^{3} (x - 1)^{3}$$

$$x^{3} + 3x^{2} + 3x + 1 = (x + 1)^{3}$$
∴ HCF = (x + 1)^{3}  
30. (D) 3748x + 5467y = 10085  
1731x + 7484y = 4034  
(-) (-) (-)  
2017x - 2017y = 6051  
2017 (x - y) = 6051  
x - y =  $\frac{6051}{2017} = 3$   
MATHEMATICS - 2 (MAQ)  
31. (A,B) a = 5, b =  $-2\sqrt{6}$  c =  $-2$   
 $x = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a}$   
 $= \frac{-(-2\sqrt{6} \pm \sqrt{(-2\sqrt{6})^{2} - 4 \times 5 \times -2})}{2(5)}$   
 $= \frac{2\sqrt{6} \pm \sqrt{24 + 40}}{10}$   
 $= \frac{2\sqrt{6} \pm 8}{10} = \frac{2(\sqrt{6} \pm 4)}{10}$   
 $= \frac{4 + \sqrt{6}}{5}$  (OR)  $\frac{-4 + \sqrt{6}}{5}$   
32. (A,B,D)  
Option (A)  
 $p(x) = 0.\overline{3}x^{2} + x - 3.\overline{3}$   
 $= (\frac{1}{3}x^{2} + x - \frac{10}{3})$   
 $p(x) = \frac{1}{3}(x^{2} + 3x - 10)$   
 $p(-5) = \frac{1}{3}[(-5)^{2} + 3(-5) - 10]$ 

$$= \frac{1}{3}(25 - 5 - 10)$$

$$p(-5) = 0 \Rightarrow (-5) \text{ is zero fo } p(x)$$

$$p(2) = \frac{1}{3}(2^{2} + 3(2) - 10)$$

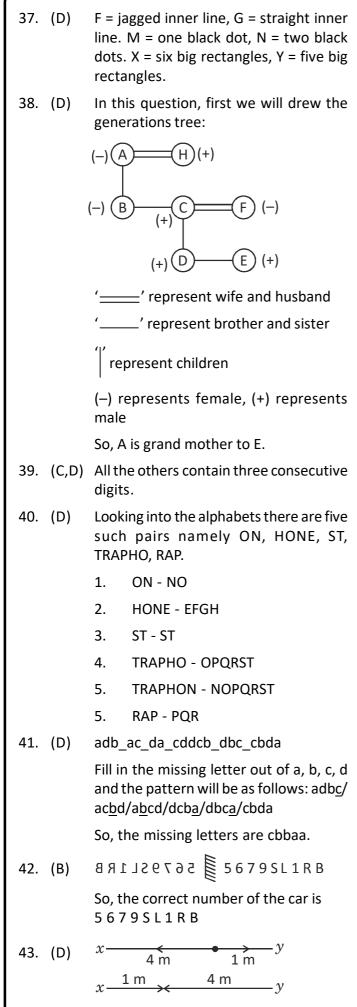
$$= \frac{1}{3}(4 + 6 - 10)$$

$$p(2) = 0 \Rightarrow '2' \text{ is the zero of } p(x)$$
33. (C,D)  $\stackrel{(-3, 2)}{\longrightarrow} \stackrel{(-3, 2)}{\longrightarrow} \stackrel{(-3$ 

$$\angle A = \frac{75^{\circ}}{2} = \left(37\frac{1}{2}\right)^{\circ}$$

$$37\frac{1}{2}^{\circ} + \angle B = 45^{\circ}$$

$$\angle B = 7\frac{1}{2}^{\circ}$$
35. (B,C)  
Given 2025, 2018, 2011, .... 1864 are in  
AP.  
a = 2025, d = a<sub>2</sub> - a<sub>1</sub> = 2018 - 20277 = -7  
a<sub>n</sub> = 1964  
a + (n + 1)d = 1864  
2025 + (n - 1) (-7) = 1864  
(n - 1)(-7) = 1864 - 2025  
(n - 1) = -\frac{161}{-7}  
n = 23 + 1 = 24  
'n' is even then middle terms are  $\left(\frac{n}{2}\right)^{\circ}$   
term and  $\left(\frac{n}{2} + 1\right)^{\circ}$  term.  
 $\therefore a_{12} = a + 11d = 2025 + 11(-7)$   
= 2025 - 77 = 1948  
 $a_{13} = a_{12} + d = 1948 + (-7)$   
= 1948 - 7 = 1941  
 $\therefore$  The middle terms are 1941 & 1948  
**REASONING**  
36. (D)  
42198  
A B C D E  
3 & 7 & 2 & 5  
C E B A D  
Similarly  
A B C D E  
9 1 4 8 2  
C E B A D  
Similarly  
A B C D E  
9 1 4 8 2  
C E B A D  
4 2 1 9 8



44.	(C)	The different symbols used by the
		mathematician to denote the operations
		are

$$> \longrightarrow '+' < \longrightarrow '-' + \longrightarrow '\div' ^ \longrightarrow 'X' - \longrightarrow '=' × \longrightarrow '>' = \longrightarrow '<' Now, consider 8 < 4 + 2 = 6 > 3 8 - 4 ÷ 2 < 6 + 3 = 6 < 9 (true) Hence option (C) follows the$$

Hence, option (C) follows the symbols correctly.

45. (B) All thieves are criminals.

Judges are different from thieves and criminals.



Option (B) is correct.

#### CRITICAL THINKING

46. (A) Let Bob take block n then the blocks you will take along with it will be n-1 and n-2. This means n divisible by 3. So. Lets try to find a number between 39 and 40 that is divisible by 3. So, according to this structure bob will taking blocks 39,38 & 37 all at once. Avvording to the given picture next numbers are 3 less than original number means 36,35,&34, but we don't need to know what is after this is because 36 is not what we need to know. So, lets going backward than as we can see this 90 is five more than 85.5 more than previous number would be 42 and this means 41 and 40 those would be the blocks underneath it. So now we have between 40 and 39 is 4 Blocks.

39	
38	
37	
42	
41	
40	

47.	(C)	Conclusion (I) : This conclusion states that religion mandates all followers to visit Mansarovar every year. The statement only mentions that thousands of pilgrims make the journey every year. It does not state that all followers are mandated to make the pilgrimage annually.		(D)	Tharun will likely look for a new job closer to home.		
					This option suggests a significant change in Tharun's life as a consequence of the train schedule change. However, the given information does not indicate that Tharun is considering such a drastic step.		
48.		Therefore, Conclusion (I) does not necessarily follow from the statement. Conclusion (II) : This conclusion states that visiting Mansarovar is an essential requirement for the salvation of all followers. The statement does not provide information about the religious significance of the pilgrimage or whether it is essential for salvation.			Conclusion : The choice that best suggests Tharun's situation based on the provided information is:		
				(B)	Tharun's commute is less comfortable since the train schedule changed.		
					This option directly reflects the impact of the new train schedule on Tharun's commute, making it the most relevant and accurate suggestion based on the given details.		
		Therefore, Conclusion (II) does not follow from the statement. Conclusion : Neither Conclusion (I) nor Conclusion (II) logically follows from the given statement.	49.	(C)	The middle of the bridge would undergo the most deflection because it is furthest away from the towers and pillars supporting the bridge. Imagine a piece of string held at either end over a gap		
	(B) (A)	Tharun would be better off taking the bus to work. This option suggests an			between two tables. The point right in the middle would drop the most.		
		alternative to the train. While it may be a logical suggestion, the given information does not indicate whether taking the bus would improve Tharun's situation or if it is even a feasible option.	50.	(B)			
	(B)	Tharun's commute is less comfortable since the train schedule changed. This option directly addresses the change in Tharun's experience due to the new train schedule. It explicitly states that Tharun's commute is now less comfortable, which aligns perfectly with the information provided.					
	(C)	Many commuters will complain about the new train schedule. This option suggests a possible reaction from other commuters. While it is likely					

from other commuters. While it is likely that other commuters are also affected, the statement focuses on Tharun's situation specifically and does not provide information about others' reactions.